

Calculation of the anomalous magnetic moment of μ in a model of electroweak-scale right-handed neutrinos

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Abstract. The lepton flavor violating process $\mu \rightarrow e\gamma$ has been previously studied in a model of electroweak-scale right-handed neutrinos. We have calculated the decay amplitude in unitarity gauge [1] and R_ξ gauge [2]. In this paper, we calculate the anomalous magnetic moment of μ in R_ξ gauge.

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1. Introduction

The Standard Model(SM) of elementary particle physics has been proved to be a remarkable successful description of high energy physics phenomena. However, the origin of neutrino mass is still an unknown fundamental problem[3, 4]. In SM neutrinos are being introduced as a zero-mass. The discovery of flavor conversion of solar and atmospheric neutrinos[5, 6] has established that neutrinos have nonzero mass and they mix among themselves, therefore providing the first evidence of new physics beyond the standard model.

On the other hand, there has been growing interest in the anomalous magnetic moments of μ during the past few years. The most recent determination of a_μ in the SM is[7]

$$a_\mu^{\text{SM}} = (11659183.4 \pm 4.9) \times 10^{-10}. \quad (1)$$

The total SM prediction of μ differs from the experimental value[8]

$$a_\mu^{\text{Exp}} = (11659208.0 \pm 6) \times 10^{-10}. \quad (2)$$

The difference between them is 3σ . Though 3σ can not be regarded as the powerful proof of the new physics, it is possible that the difference will become larger with the development of experiments. So, we maybe get the proof of the new physics later.

A model with the above desired feature built in has been suggested recently by Hung [9]. The model keeps the SM gauge group albeit in a ‘vector-like’ manner: Mirror fermions charged under SM gauge group, of which right-handed neutrinos are a member, and two Higgs triplets in a way that preserves the ρ parameter to unity, plus a Higgs singlet. In particular, a right-handed neutrino which is sterile and are required to have a mass now becomes a member of a weak doublet of mirror leptons. A tiny Dirac mass for neutrinos is offered by a scalar singlet whose vacuum expectation value is not necessarily associated with the electroweak scale, while a Majorana mass of order the electroweak scale is provided by a scalar triplet. It is conceivable that these new leptons could be discovered at high energy colliders in the near future and the rich lepton flavor structure could be observed in low energy processes. The weak charged couplings are generally non-unitary with or without restricting to the subspace of light leptons, and flavor changing neutral currents (FCNC) occur in a way that is controlled by the weak charged couplings.

The paper is organized as follows. First, we will introduce the Hung model briefly; for a full account of the model, we refer to refs [9]. Then we show in section 3 how to calculate the anomalous magnetic moment of μ in the virtual transition $\mu \rightarrow \mu\gamma$ in R_ξ gauge, and demonstrate the final result. Our result is summarized in the last section.

2. Hung’s Model

We start with a brief description of the model relevant to our later analysis; for a full account of it, see Ref.[9]. Considering three generations, the SM and mirror leptons with quantum numbers under the gauge group $SU(2) \times U(1)_Y$ are:

$$F_L = \begin{pmatrix} n_L \\ f_L \end{pmatrix} (2, Y = -1), \quad f_R (1, Y = -2);$$

$$F_R^M = \begin{pmatrix} n_R^M \\ f_R^M \end{pmatrix} (2, Y = -1), f_L^M (1, Y = -2); \quad (3)$$

where the subscripts L, R refer to chirality and the superscript M to mirror, and the first number in parentheses stands for the dimension of representation under the gauge group $SU(2)$. Because of anomaly cancelation, the quark sector also has mirror partners which are of no interest here. Besides the SM scalar doublet Φ , the model contains the new scalars

$$\phi (1, 0), \chi (3, 2), \quad (4)$$

plus an additional triplet $\xi (3, 0)$ that together with χ preserves the custodial symmetry [10] but is irrelevant here.

The Yukawa couplings of leptons are, with the generation indices suppressed,

$$\begin{aligned} -\mathcal{L}_\Phi &= y \overline{F_L} \Phi f_R + y_M \overline{F_R^M} \Phi f_L^M + \text{h.c.}, \\ -\mathcal{L}_\phi &= x_F \overline{F_L} F_R^M \phi + x_f \overline{f_R} f_L^M \phi + \text{h.c.}, \\ -\mathcal{L}_\chi &= \frac{1}{2} z_M \overline{(F_R^M)^C} (i\tau^2) \chi F_R^M + \text{h.c.} \end{aligned} \quad (5)$$

where $\psi^C = \mathcal{C} \gamma^0 \psi^*$, $\mathcal{C} = i\gamma^0 \gamma^2$, and

$$\chi = \frac{1}{\sqrt{2}} \vec{\tau} \cdot \vec{\chi} = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi^+ & \sqrt{2} \chi^{++} \\ \sqrt{2} \chi^0 & -\chi^+ \end{pmatrix}. \quad (6)$$

Note that a potential Majorana coupling of χ to F_L is forbidden by imposing an appropriate $U(1)$ symmetry. Suppose the VEVs have the structure:

$$\langle \Phi \rangle = \frac{v_2}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \langle \phi \rangle = v_1, \langle \chi \rangle = v_3 \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad (7)$$

where $v_{2,3}$ contribute to the masses of weak gauge bosons and are naturally of order the electroweak scale while v_1 is not necessarily related to it. In the basis of f, f^M , the charged lepton mass terms are

$$\begin{aligned} -\mathcal{L}_m^f &= (\overline{f_L}, \overline{f_L^M}) m_f \begin{pmatrix} f_R \\ f_R^M \end{pmatrix} + \text{h.c.}, \\ m_f &= \begin{pmatrix} \frac{v_2}{\sqrt{2}} y & v_1 x_F \\ v_1 x_f^\dagger & \frac{v_2}{\sqrt{2}} y_M^\dagger \end{pmatrix}. \end{aligned} \quad (8)$$

while the neutrino mass terms are

$$\begin{aligned} -\mathcal{L}_m^n &= \frac{1}{2} (\overline{n_L}, \overline{(n_R^M)^C}) m_n \begin{pmatrix} n_L^C \\ n_R^M \end{pmatrix} + \text{h.c.}, \\ m_n &= \begin{pmatrix} 0 & v_1 x_F \\ v_1 x_F^T & v_3 z_M \end{pmatrix}. \end{aligned} \quad (9)$$

We denote the mass eigenstate fields of the charged and neutral leptons by ℓ_j , ν_j respectively, with $j = 1, 2, 3$ for those that are mostly ordinary leptons and $j = 4, 5, 6$ for

those that are mostly mirror. The seesaw mechanism operates for a Majorana mass of order the electroweak scale and Dirac proportional to v_1 which can be chosen small. This relaxes in some sense the tension in ordinary seesaw models between the generation of light neutrino mass and the observability of heavy neutrinos at colliders. The mass matrices are diagonalized by unitary transformations as follows

$$\begin{pmatrix} f_{L,R} \\ f_{L,R}^M \end{pmatrix} = X_{L,R} \ell_{L,R}, \quad \begin{pmatrix} n_L^C \\ n_R^M \end{pmatrix} = Y \nu_R, \quad \begin{pmatrix} n_L \\ (n_R^M)^C \end{pmatrix} = Y^* \nu_L, \quad \nu_L = \nu_R^C, \quad (10)$$

where the unitary matrices $X_{L,R}$ and Y satisfy

$$X_L^\dagger m_f X_R = m_\ell = \text{diag}(m_{\ell_i}), \quad Y^T m_n Y = m_\nu = \text{diag}(m_{\nu_i}), \quad (11)$$

with the mass eigenvalues m_{ℓ_i, ν_i} being real and nonnegative. There is a constraint on their masses from the zero texture,

$$\sum_{k=1}^6 m_{\nu_i} Y_{ik} Y_{jk} = 0, \quad \text{for } i, j = 1, 2, 3. \quad (12)$$

Splitting each unitary matrix into two blocks that contain the upper and lower rows respectively,

$$X_{L,R} = \begin{pmatrix} X_{L,R}^u \\ X_{L,R}^d \end{pmatrix}, \quad Y = \begin{pmatrix} Y^u \\ Y^d \end{pmatrix}, \quad (13)$$

the following relations will be required later:

$$f_{L,R} = X_{L,R}^u \ell_{L,R}, \quad f_{L,R}^M = X_{L,R}^d \ell_{L,R}, \quad n_L = Y^{u*} \nu_L, \quad n_R^M = Y^d \nu_R, \quad (14)$$

where the unitarity constraints become

$$X_\alpha^u X_\alpha^{u*} = X_\alpha^d X_\alpha^{d*} = 1_3, \quad X_\alpha^u X_\alpha^{d+} = 0_3 \quad (\text{for } \alpha = L, R), \\ Y^u Y^{u+} = Y^d Y^{d+} = 1_3, \quad Y^u Y^{d+} = 0_3, \quad (15)$$

and the zero texture constraint only acts on Y^u :

$$Y^u m_\nu Y^{uT} = 0_3. \quad (16)$$

The above diagonalizing matrices will enter the gauge interactions of leptons. Some algebra yields,

$$\mathcal{L}_g = g_2 \left(j_W^{+\mu} W_\mu^+ + j_W^{-\mu} W_\mu^- + J_Z^\mu Z_\mu \right) + e J_{\text{em}}^\mu A_\mu, \quad (17)$$

where the currents are ($P_{L,R} = 1 \mp \gamma_5/2$)

$$\begin{aligned} \sqrt{2} j_W^{+\mu} &= \bar{\nu} \gamma^\mu (V_L P_L + V_R P_R) \ell, \\ c_W J_Z^\mu &= \frac{1}{2} \bar{\nu} \gamma^\mu (V_L V_L^\dagger P_L + V_R V_R^\dagger P_R) \nu - \frac{1}{2} \bar{\ell} \gamma^\mu (V_L^\dagger V_L P_L + V_R^\dagger V_R P_R) \ell \\ &\quad + s_W^2 \bar{\ell} \gamma^\mu \ell, \\ J_{\text{em}}^\mu &= -\bar{\ell} \gamma^\mu \ell, \end{aligned} \quad (18)$$

with $v = v_R + v_L = v_R + v_R^C = v_L + v_L^C$, and $c_W = \cos \theta_W, s_W = \sin \theta_W$ with θ_W being the Weinberg angle. To relate the matrices V_L, V_R to X_a ($a = L, R$), Y , it is convenient to decompose the latter into the up and down 3×6 blocks,

$$X_a = \begin{pmatrix} X_a^u \\ X_a^d \end{pmatrix}, Y = \begin{pmatrix} Y^u \\ Y^d \end{pmatrix}, \quad (19)$$

then

$$V_L = Y^{uT} X_L^u, V_R = Y^{d\dagger} X_R^d, \quad (20)$$

with $V_L^T V_R = 0$. These matrices are generally non-unitary and the deviation from unitarity induces FCNC in both sectors of neutrinos and charged leptons:

$$\begin{aligned} V_L V_L^\dagger &= Y^{uT} Y^{u*}, V_R V_R^\dagger = Y^{d\dagger} Y^d, \\ V_L^\dagger V_L &= X_L^{u\dagger} X_L^u, V_R^\dagger V_R = X_R^{d\dagger} X_R^d. \end{aligned} \quad (21)$$

The matrix blocks X_L^d, X_R^u do not enter charged current interactions since f_L^M, f_R are $SU(2)$ singlets. It is important to note that the matrices in charged currents, V_L, V_R , are generally not unitary and there are flavor changing neutral currents arising from the nonunitarity.

If we go on with our calculations, we will also need the Yukawa couplings of the would-be Goldstone bosons (GB's). Although the original scalar fields mix in a complicated manner via the terms in the scalar potential, GB's are independent of mixing details due to gauge symmetry and their Yukawa couplings only involve matrices that already appear in gauge interactions. We list the Feynman rules as follows:

$$\begin{aligned} \bar{\nu}_i \ell_\alpha G^+ &: +i \frac{g^2}{\sqrt{2} m_W} [m_i (V_{i\alpha}^L P_L + V_{i\alpha}^R P_R) - m_\alpha (V_{i\alpha}^L P_R + V_{i\alpha}^R P_L)], \\ \bar{\ell}_\alpha \nu_i G^- &: +i \frac{g^2}{\sqrt{2} m_W} [m_i (V_{i\alpha}^{L*} P_R + V_{i\alpha}^{R*} P_L) - m_\alpha (V_{i\alpha}^{L*} P_L + V_{i\alpha}^{R*} P_R)]. \end{aligned} \quad (22)$$

We give below other Feynman rules that will be required later (all momenta incoming):

$$\begin{aligned} A_\mu G^+(k_+) G^-(k_-) &: i^2 e(-i)(k_+ - k_-)_\mu = ie(k_+ - k_-)_\mu, \\ A_\mu W_\nu^\pm G^\mp &: i e m_W g_{\mu\nu}, \\ A_\mu(k) W_\alpha^+(k_+) W_\beta^-(k_-) &: \Gamma_{\alpha\beta\mu} = g_{\alpha\beta}(k_- - k_+)_\mu + g_{\beta\mu}(k - k_-)_\alpha \\ &+ g_{\mu\alpha}(k_+ - k)_\beta. \end{aligned} \quad (23)$$

3. Calculation in R_ξ gauge

Now we begin to calculate the anomalous magnetic moment for μ in both W diagram and Z diagram. As discussed in [2], there are two types of contributions at the one loop level that are mediated respectively by charged current interactions of W^\pm and flavor changing neutral current interactions of Z^0 , corresponding to unitarity gauge and R_ξ gauge. For the charged current diagrams we find that they involve a triple gauge coupling which is more divergent in the ultraviolet and the flavor mixing matrices in the charged current are not unitary. Thus in such a circumstance, it is highly desired that the calculation should be done in a safer R_ξ gauge whose ξ dependence is canceled as expected.

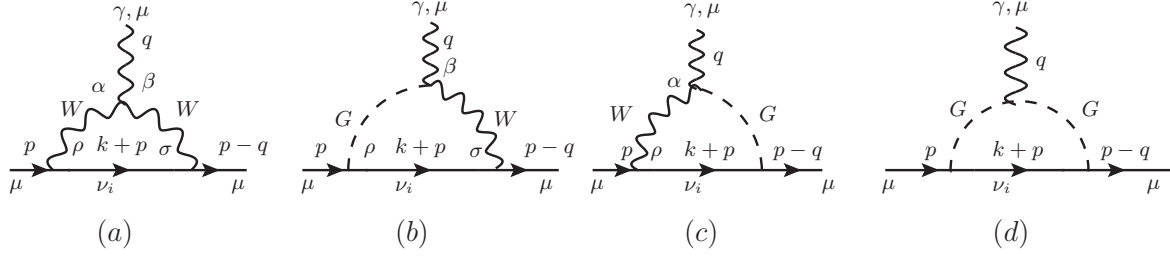


Figure 1. Diagrams that contribute to the decay $\mu \rightarrow \mu \gamma$.

3.1. The contribution of W diagram

First of all, we should calculate the amplitude for the decay $\mu(p) \rightarrow \mu(p-q)\gamma(q, \epsilon)$ in W diagram whose Feynman diagrams are shown in Fig 1. The anomalous magnetic moment of μ is:

$$a_\mu = a_\mu^W + a_\mu^Z \equiv \frac{g-2}{2} = F_2(q^2=0). \quad (24)$$

where

$$\Gamma^\mu(p', p) \sim \frac{i\sigma_{\mu\nu}q^\nu}{2m_\mu} F_2(q^2). \quad (25)$$

The Lorentz and gauge symmetries dictate that the amplitude has the following structure [11]:

$$\mathcal{A} \sim \bar{u}_\mu i\sigma_{\lambda\nu} q^\nu [x + y\gamma_5] \epsilon^\lambda u_\mu, \quad (26)$$

so that we can concentrate on the $i\sigma_{\lambda\nu} q^\nu$ terms to pick up the coefficients, x and y .

We shall keep only terms up to the linear order in the muon mass, m_μ . These are indeed very good approximations. The four diagrams give the following on-shell amplitudes; for a full presentation of our calculation details, see Appendix:

$$\begin{aligned} T(a) = eg_2^2 \bar{u}_\mu (i\sigma_{\mu\nu} q^\nu) \frac{i}{(4\pi)^2 m_W^2} \Big\{ & m_\mu A \left[M(a) + P(a) \ln[r_i] + Q(a) \ln[\xi] \right] \\ & + m_i B \left[M'(a) + P'(a) \ln[r_i] + Q'(a) \ln[\xi] \right] \\ & + m_\mu B \left[M''(a) + P''(a) \ln[r_i] \right] \\ & + m_\mu D \left[M_D(a) + P_D(a) \ln[r_i] + Q_D(a) \ln[\xi] \right] \Big\} u_\mu, \end{aligned} \quad (27)$$

$$\begin{aligned} T(b) = eg_2^2 \bar{u}_\mu (i\sigma_{\mu\nu} q^\nu) \frac{i}{(4\pi)^2 m_W^2} \Big\{ & m_\mu A \left[M(b) + P(b) \ln r_i + Q(b) \ln \xi \right] \\ & + m_i B \left[M'(b) + P'(b) \ln r_i + Q'(b) \ln \xi \right] \\ & + m_\mu D \left[M_D(b) + P_D(b) \ln r_i + Q_D(b) \ln \xi \right] \Big\} u_\mu, \end{aligned} \quad (28)$$

$$\begin{aligned} T(c) = eg_2^2 \bar{u}_\mu (i\sigma_{\mu\nu} q^\nu) \frac{i}{(4\pi)^2 m_W^2} \Big\{ & m_\mu A \left[M(c) + P(c) \ln r_i + Q(c) \ln \xi \right], \\ & + m_i B \left[M'(c) + P'(c) \ln r_i + Q'(c) \ln \xi \right] + m_\mu D \left[M_D(c) + P_D(c) \ln r_i \right] \end{aligned}$$

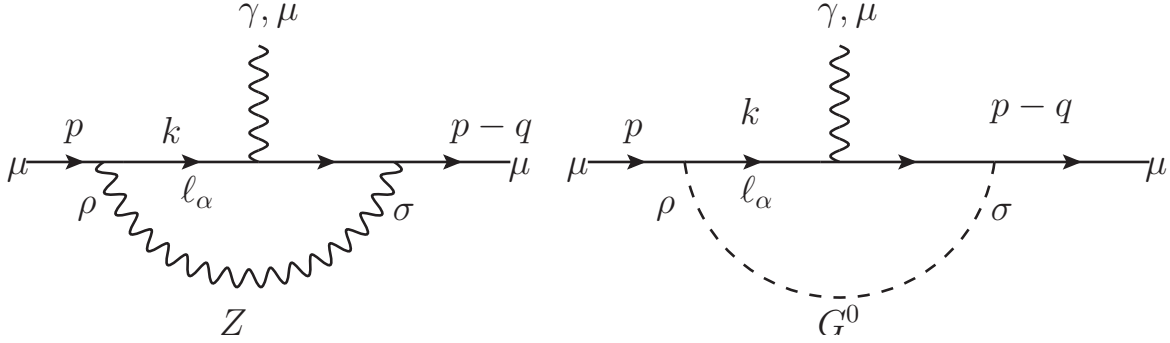


Figure 2. Diagrams that contribute to the decay $\mu \rightarrow \mu \gamma$.

$$+Q_D(c) \ln \xi \Big] \Big\} u_\mu, \quad (29)$$

$$\begin{aligned} T(d) = & eg_2^2 \bar{u}_\mu (i\sigma_{\mu\nu} q^\nu) \frac{i}{(4\pi)^2 m_W^2} \Big\{ m_\mu A \Big[M(d) + P(d) \ln r_i + Q(d) \ln \xi \Big] \\ & + m_i B \Big[M'(d) + P'(d) \ln r_i + Q'(d) \ln \xi \Big] \\ & + m_\mu D \Big[M_D(d) + P_D(d) \ln r_i + Q_D(d) \ln \xi \Big] \Big\} u_\mu. \end{aligned} \quad (30)$$

The loop functions such as $M(a), M(b), M(c), M(d), \dots$ and other functions are listed in Appendix.

Collecting the contributions from the four graphs yields the final answer:

$$\begin{aligned} a_\mu^W = & \frac{\sqrt{2} G_F}{(4\pi)^2} \Big[m_\mu^2 A_W \mathcal{F}_W(r_i) + m_\mu m_i B_W \mathcal{G}_W(r_i) + m_\mu^2 B_W \mathcal{P}_W(r_i) + m_\mu^2 D_W \mathcal{Q}_W(r_i) \Big], \\ A_W = & \sum_i (V_{Li\mu}^* V_{Li\mu} + V_{Ri\mu}^* V_{Ri\mu}) = D_W, \\ B_W = & \sum_i (V_{Li\mu}^* V_{Ri\mu} + V_{Ri\mu}^* V_{Li\mu}). \end{aligned} \quad (31)$$

where $\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8m_W^2}$. The loop functions are:

$$\begin{aligned} \mathcal{F}_W = & \frac{1}{6(1-r_i)^4} (10 - 43r_i + 78r_i^2 - 49r_i^3 + 4r_i^4 + 18r_i^3 \ln r_i), \\ \mathcal{G}_W = & \frac{1}{(1-r_i)^3} (-4 + 15r_i - 12r_i^2 + r_i^3 + 6r_i^2 \ln r_i), \\ \mathcal{P}_W = & \frac{1}{(1-r_i)^3} [13r_i^2 - 24r_i + 11 + 2r_i(5 - 6r_i) \ln r_i], \\ \mathcal{Q}_W = & \frac{1}{6(1-r_i)^4} [7 - 34r_i + 33r_i^2 - 10r_i^3 + 4r_i^4 - 18r_i^2 \ln r_i]. \end{aligned} \quad (32)$$

It is good to see that the ξ dependence is completely cancelled as expected.

3.2. The contribution of Z diagram

We begin to calculate the amplitude of diagram Z whose Feynman diagrams are shown in Fig 2. We can use the same method in the diagram W. Thus we can give the contributions from the two graphs in Z diagram; for a full presentation of our calculation details, see Appendix:

$$T[(b)_Z] = \frac{eg_2^2}{4c_W m_Z^2} \frac{i}{(4\pi)^2} \bar{u}_\mu(i\sigma_{\mu\nu} q^\nu) \left\{ m_\mu A' [MM(b) + PP(b) \ln r_\alpha + QQ(b) \ln \xi] \right. \\ \left. + m_\alpha B' [MM'(b) + PP'(b) \ln r_\alpha + QQ'(b) \ln \xi] \right. \\ \left. + m_\mu D' [MM_D(b) + PP_D(b) \ln r_\alpha + QQ_D(b) \ln \xi] \right\} u_\mu, \quad (33)$$

$$T[b_{G^0}] = \frac{eg_2^2}{4c_W m_Z^2} \frac{i}{(4\pi)^2} \bar{u}_\mu(i\sigma_{\mu\nu} q^\nu) \left\{ m_\mu A' [MM(c) + PP(c) \ln r_\alpha + QQ(c) \ln \xi] \right. \\ \left. + m_\alpha B' [MM'(c) + PP'(c) \ln r_\alpha + QQ'(c) \ln \xi] \right. \\ \left. + m_\mu D' [MM_D(c) + PP_D(c) \ln r_\alpha + QQ_D(c) \ln \xi] \right\} u_\mu, \quad (34)$$

Using (31) and (32), we finally get the contribution of the diagram Z to the anomalous magnetic moment of μ :

$$a_\mu^Z = \frac{\sqrt{2}G_{FCW}}{(4\pi)^2} \left[m_\mu^2 A'_Z \mathcal{F}_Z(r_\alpha) + m_\mu m_\alpha B'_Z \mathcal{G}_Z(r_\alpha) + m_\mu^2 B'_Z \mathcal{P}_Z(r_\alpha) + m_\mu^2 D'_Z \mathcal{Q}_Z(r_\alpha) \right], \\ A'_Z = \sum_\alpha (c_{\alpha\mu}^{L*} c_{\alpha\mu}^L + c_{\alpha\mu}^{R*} c_{\alpha\mu}^R) = D'_Z, \\ B'_Z = \sum_\alpha (c_{\alpha\mu}^{L*} c_{\alpha\mu}^R + c_{\alpha\mu}^{R*} c_{\alpha\mu}^L). \quad (35)$$

where

$$\mathcal{F}_Z = \frac{1}{4(1-r_\alpha)^4} [-2 + 2r_\alpha - r_\alpha^2 + 10r_\alpha^3 - 9r_\alpha^4 + 2r_\alpha^2(-5 + 8r_\alpha) \ln r_\alpha], \\ \mathcal{G}_Z = \frac{1}{2(1-r_\alpha)^3} [-4 - r_\alpha + 8r_\alpha^2 - 3r_\alpha^3 - 6r_\alpha^2 \ln r_\alpha], \\ \mathcal{P}_Z = \frac{1}{(1-r_\alpha)^3} [6r_\alpha^2 - 13r_\alpha + 11 + 6r_\alpha(5 - 6r_\alpha) \ln r_\alpha], \\ \mathcal{Q}_Z = \frac{1}{12(1-r_\alpha)^4} [-14 - 6r_\alpha + 57r_\alpha^2 - 54r_\alpha^3 + 18r_\alpha^4 - 6r_\alpha^2 \ln r_\alpha]. \quad (36)$$

We find the result is also nothing to do with ξ as expected.

4. Numerical analysis

We have calculated the W and Z diagrams in R_ξ gauge. An interesting technical point is in order. We can also work in unitarity gauge and it should be simplest. But in the W diagram there is more ultraviolet divergent due to the triple gauge coupling. Thus there is no guarantee in this case that the order of removing the ultraviolet regulator commutes with that of taking the unitarity gauge limit. As a matter of fact, although the diagram is convergent in both unitarity and R_ξ gauges, there is a finite difference in the terms linear in the lepton masses

Table 1. The anomalous magnetic moment of $\mu(a_\mu \times 10^{-13})$ is increasing with the increase of the heavy neutrino mass when the unknown α mass is a fixed value at first, while it is decreasing when the mass of the heavy neutrino is over 700 GeV.

$m_\alpha(\text{GeV}) \backslash m_i^h(\text{GeV})$	200	300	400	500	600	700	800	900
200	4.58086	4.58396	4.58459	4.58476	4.58481	4.58482	4.58481	4.58480
300	5.68684	5.68994	5.69056	5.69074	5.69079	5.69080	5.69079	5.69078
400	6.94412	6.94722	6.94784	6.94802	6.94807	6.94807	6.94807	6.94805
500	8.26680	8.26990	8.27052	8.27070	8.27075	8.27075	8.27075	8.27073
600	9.62381	9.62691	9.62753	9.62771	9.62776	9.62777	9.62776	9.62775
700	11.0017	11.0048	11.0054	11.0056	11.0056	11.0056	11.0056	11.0056
800	12.3934	12.3965	12.3971	12.3973	12.3973	12.3974	12.3973	12.3973
900	13.7950	13.7981	13.7987	13.7989	13.7989	13.7989	13.7989	13.7989

between the results obtained in the two gauges. This caveat is restricted to the mentioned terms because terms of a higher order are convergent enough to allow the free interchange of taking the limits. Considering this, we should work in R_ξ gauge make sure not to have any problems.

Using (29) and (33), we finally obtain the anomalous magnetic moment of μ :

$$a_\mu = a_\mu^W + a_\mu^Z \equiv \frac{g-2}{2} = F_2(q^2=0). \quad (37)$$

The above anomalous magnetic moment involves several neutrino masses, the unknown α mass and many mixing matrix elements. In our later numerical analysis, we shall make some approximations. First, the light neutrinos can be safely treated as massless in the diagram W. Then, $\mathcal{F}_W = \mathcal{F}_W^l \rightarrow \frac{5}{3}$, $\mathcal{P}_W = \mathcal{P}_W^l \rightarrow 11$, $\mathcal{Q}_W = \mathcal{Q}_W^l \rightarrow \frac{7}{6}$. The term \mathcal{G} can be dropped because of m_i . Second, for the heavy neutrino mass we choose $m_h = 200, 300$ up to 900 GeV. We use $m_W = 80.2$ GeV. Thus $r_i = 6.21887$. Then, $\mathcal{F}_W = \mathcal{F}_W^h \rightarrow 0.0883211$, $\mathcal{G}_W = \mathcal{G}_W^h \rightarrow -0.350906$, $\mathcal{P}_W = \mathcal{P}_W^h \rightarrow -0.320226$, $\mathcal{Q}_W = \mathcal{Q}_W^h \rightarrow 0.920466$.

As a bonus of the approximations, the anomalous magnetic moment of μ in the diagram W depends on the products of matrix elements summed over light and heavy neutrinos respectively:

$$\begin{aligned} V_1^l &= \sum_{i=1}^3 (V_L^\dagger)_{ei} (V_L)_{i\mu}, \quad V_2^l = \sum_{i=1}^3 (V_R^\dagger)_{ei} (V_R)_{i\mu}, \\ V_3^l &= \sum_{i=1}^3 (V_R^\dagger)_{ei} (V_L)_{i\mu}, \quad V_4^l = \sum_{i=1}^3 (V_L^\dagger)_{ei} (V_R)_{i\mu}, \end{aligned} \quad (38)$$

and similarly for $V_{1,2,3,4}^h$ with i summed over 4, 5, 6.

We find an algebraically simple case after some inspection. Suppose the upper-right 3×3 block of Y is real. In this scenario, our special neutrino spectrum (three almost massless plus three almost degenerate and heavy) implies that the two off-diagonal 3×3 blocks of Y vanish, the lower-right block is trivially identity and the upper-left one is unitary. Then, $V_1^l = (x_L^\dagger x_L)_{e\mu}$, $V_2^h = -(x_R^\dagger x_R)_{e\mu}$ while all others vanish, where $x_{L,R}$ are the upper-left 3×3

blocks of $X_{L,R}$ respectively. Since we have no idea of their magnitudes, we sample randomly the real and imaginary parts of V_1^l, V_2^h between -2×10^{-6} and $+2 \times 10^{-6}$.

Thus for numerical analysis we can assume $A_W, D_W \sim 10^{-6}, B_W \sim 10^{-6}$ with i summed over 1,2,3 while $B_W = 0$ with i summed over 4,5,6. We also use $\alpha = 1/137.04, m_\mu = 0.1056$ GeV.

We proceed to the diagram Z. First, we can also assume the unknown α mass $m_\alpha = 200, 300$ up to 900 GeV. Then, $r_\alpha = 4.81048$. We use $m_Z = 91.187621$ GeV from PDG. Thus, we get: $\mathcal{F}_Z \rightarrow -3.15976, \mathcal{G}_Z \rightarrow 2.28058, \mathcal{P}_Z \rightarrow 6.91432, \mathcal{Q}_Z \rightarrow 1.90094$. We also make some approximations:

$$\begin{aligned} \left(c_{\alpha\mu}^{L*} c_{\alpha\mu}^L \right)_{\ll} &\approx 2(V_L^\dagger V_L)_{\alpha\mu}^* (V_L^\dagger V_L)_{\alpha\mu} - 4s_W^2 (V_L^\dagger V_L)_{\alpha\mu} \sim 2(1 - 2s_W^2)10^{-6} \\ &\sim 10^{-7} \sim \left(c_{\alpha\mu}^{R*} c_{\alpha\mu}^R \right)_{\ll} \sim \left(c_{\alpha\mu}^{L*} c_{\alpha\mu}^R \right)_{\ll}, \end{aligned} \quad (39)$$

Thus we obtain the different value of the anomalous magnetic moment of μ which has been shown in Table 1.

It is interesting that with the fixed unknown m_α the anomalous magnetic moment of μ is increasing with the increase of the heavy neutrino mass at first, while it is decreasing when the mass of the heavy neutrino is over 700 GeV.

5. Conclusion

The subject of the anomalous magnetic moments for the muon is an exciting and fascinating topic because it represents the best compromise between sensitivity to new degrees of freedom describing physics beyond the standard model and experimental feasibility. We have calculated in R_ξ gauge the anomalous magnetic moment of μ in a model suggested recently. Because of the rich flavor structure of the model, the weak charged currents involve nonunitary mixings between the neutral and charged leptons and contain both left-handed and right-handed chiralities. It is conceivable that these new leptons could be discovered at high energy colliders in the near future, while the rich lepton flavor structure could be observed in low energy processes.

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Appendix A. The calculation of the contribution from W diagram

We start with diagram (b):

$$(b) = \frac{eg_2^2}{2} \bar{u}_\mu \int \frac{d^4 k}{(2\pi)^4} \frac{N_\mu \left[m_i \left((V_L)_{i\mu} P_L + (V_R)_{i\mu} P_R \right) - m_\mu \left((V_L)_{i\mu} P_R + (V_R)_{i\mu} P_L \right) \right]}{[k^2 - \xi m_W^2][(k+q)^2 - m_W^2]} u_\mu, \quad (\text{A.1})$$

where

$$N_\mu = \gamma_\sigma \left[(V_L^\dagger)_{\mu i} P_L + (V_R^\dagger)_{\mu i} P_R \right] \frac{1}{\not{k} + \not{p} - m_i} \left(g_\mu^\sigma - (1 - \xi) \frac{(k+q)_\mu (k+q)^\sigma}{(k+q)^2 - \xi m_W^2} \right). \quad (\text{A.2})$$

To simplify N_μ , we apply $\bar{u}_\mu \not{q} = \bar{u}_\mu (-m_\mu + \not{p})$; i.e., we can make the replacement, $\not{q} \rightarrow \not{p} - m_\mu$, when \not{q} is on the far left. Note that the term q_μ has no contribution. Now N_μ can be split up as follows :

$$N_\mu = \text{I} + \text{II} + \text{III} + \text{VI}, \quad (\text{A.3})$$

where

$$\begin{aligned} \text{I} &= \left[(V_L^\dagger)_{\mu i} P_R + (V_R^\dagger)_{\mu i} P_L \right] \gamma_\mu \frac{\not{k} + \not{p} + m_i}{k^2 + p^2 - m_i^2}, \\ \text{II} &= -(1 - \xi) \left[(V_L^\dagger)_{\mu i} P_R + (V_R^\dagger)_{\mu i} P_L \right] \frac{k_\mu}{(k+q)^2 - \xi m_W^2}, \\ \text{III} &= -(1 - \xi) (V_L^\dagger P_R + V_R^\dagger P_L) m_i \frac{k_\mu (\not{k} + \not{p} + m_i)}{[(k+p)^2 - m_i^2][(k+q)^2 - \xi m_W^2]}, \\ \text{IV} &= (1 - \xi) (V_L^\dagger P_L + V_R^\dagger P_R) m_\mu \frac{k_\mu (\not{k} + \not{p} + m_i)}{[(k+p)^2 - m_i^2][(k+q)^2 - \xi m_W^2]}. \end{aligned} \quad (\text{A.4})$$

These three terms will be calculated separately below.

The term I gives the following contribution to the loop integral:

$$\begin{aligned} (b)_\text{I} &= \frac{eg_2^2}{2} \bar{u}_\mu \int \frac{d^4 k}{(2\pi)^4} \frac{\left[(V_L^\dagger)_{\mu i} P_R + (V_R^\dagger)_{\mu i} P_L \right] \gamma_\mu (\not{k} + \not{p} + m_i)}{[k^2 + p^2 - m_i^2][k^2 - \xi m_W^2][(k+q)^2 - m_W^2]} \\ &\quad \times \left[m_i \left((V_L)_{i\mu} P_L + (V_R)_{i\mu} P_R \right) - m_\mu \left((V_L)_{i\mu} P_R + (V_R)_{i\mu} P_L \right) \right] u_\mu, \end{aligned} \quad (\text{A.5})$$

As mentioned before, we only need pick up the $(i\sigma_{\mu\nu} q^\nu)$ terms. Using $\not{p} u_\mu = m_\mu u_\mu$, we can make the replacement $\not{p} \rightarrow m_\mu$ when \not{p} is on the far right. Since we work up to the linear order in m_μ , it is sufficient to expand to $O(p^2)$. The expansion yields several types of terms. By symmetric integration, we get $\gamma_\mu \not{k} (-2k \cdot p) \rightarrow -\frac{1}{2} \gamma_\mu \not{p} k^2$ which has no contribution to the desired Lorentz structure. Similarly, the other two terms are, $\gamma_\mu \not{k} (-2k \cdot q) \rightarrow -\frac{1}{2} \gamma_\mu \not{q} k^2 \rightarrow \frac{1}{2} (i\sigma_{\mu\nu} q^\nu) k^2$. The contribution of the I term is summarized as follows:

$$\begin{aligned} (b)_\text{I} &= \frac{1}{4} eg_2^2 \bar{u}_\mu (i\sigma_{\mu\nu} q^\nu) m_i B \int_k \frac{k^2}{[k^2 - m_i^2][k^2 - \xi m_W^2][k^2 - m_W^2]^2} u_\mu \\ &\quad - \frac{1}{4} eg_2^2 \bar{u}_\mu (i\sigma_{\mu\nu} q^\nu) m_\mu A \int_k \frac{k^2}{[k^2 - m_i^2][k^2 - \xi m_W^2][k^2 - m_W^2]^2} u_\mu, \end{aligned} \quad (\text{A.6})$$

where

$$\begin{aligned}\int_k &\equiv \int \frac{d^4 k}{(2\pi)^4}, \\ A &= \sum_i \left((V_L^\dagger)_{\mu i} (V_L)_{i\mu} P_R + (V_R^\dagger)_{\mu i} (V_R)_{i\mu} P_L \right), \\ B &= \sum_i \left((V_L^\dagger)_{\mu i} (V_R)_{i\mu} P_R + (V_R^\dagger)_{\mu i} (V_L)_{i\mu} P_L \right).\end{aligned}\tag{A.7}$$

We find that the term II has no contribution to (b).

In calculating the term III, we will use:

$$\begin{aligned}& k_\mu \not{k} (k \cdot p) (k \cdot q) \\ & \rightarrow \frac{1}{48} (i\sigma_{\mu\nu} q^\nu) \not{p} k^4 - \frac{1}{48} (i\sigma_{\mu\nu} q^\nu) m_\mu^* k^4, \\ & k_\mu \not{k} (k \cdot p) (k \cdot p) \\ & \rightarrow \frac{1}{24} (i\sigma_{\mu\nu} q^\nu) \not{p} k^4.\end{aligned}\tag{A.8}$$

Note that adding the subscript * to m_μ means $P_L \rightarrow P_R, P_R \rightarrow P_L$ when P_L and P_R are on the far left. Thus we get the contribution of the term III to the loop integral:

$$\begin{aligned}(b)_{\text{III}} &= \frac{1}{24} e g_2^2 \bar{u}_\mu (i\sigma_{\mu\nu} q^\nu) m_i^2 m_\mu A (1 - \xi) \\ &\quad \times \int_k k^4 \left[-\frac{1}{(k^2 - m_i^2)^2 (k^2 - \xi m_W^2)^3 (k^2 - m_W^2)} \right. \\ &\quad \left. - \frac{1}{(k^2 - m_i^2)^2 (k^2 - \xi m_W^2)^2 (k^2 - m_W^2)^2} \right. \\ &\quad \left. - 2 \frac{1}{(k^2 - m_i^2)^3 (k^2 - \xi m_W^2)^2 (k^2 - m_W^2)} \right] u_\mu \\ &\quad + \frac{1}{8} e g_2^2 \bar{u}_\mu (i\sigma_{\mu\nu} q^\nu) m_i^3 B (1 - \xi) \int_k \frac{k^2}{(k^2 - m_i^2)^2 (k^2 - \xi m_W^2)^2 (k^2 - m_W^2)} u_\mu \\ &\quad + e g_2^2 \bar{u}_\mu (i\sigma_{\mu\nu} q^\nu) (1 - \xi) m_i^2 m_\mu D \int_k \left[\frac{1}{24} \frac{k^4}{(k^2 - m_i^2)^2 (k^2 - \xi m_W^2)^3 (k^2 - m_W^2)} \right. \\ &\quad \left. + \frac{1}{24} \frac{k^4}{(k^2 - m_i^2)^2 (k^2 - \xi m_W^2)^2 (k^2 - m_W^2)^2} \right] u_\mu.\end{aligned}\tag{A.9}$$

We can use the same method in the term IV and we get the contribution of it:

$$(b)_{\text{IV}} = e g_2^2 \bar{u}_\mu (i\sigma_{\mu\nu} q^\nu) (1 - \xi) m_i^2 m_\mu D \left(-\frac{1}{8} \right) \frac{k^2}{(k^2 - m_i^2)^2 (k^2 - \xi m_W^2)^2 (k^2 - m_W^2)} u_\mu.\tag{A.10}$$

where

$$D = \sum_i \left((V_L^\dagger)_{\mu i} (V_L)_{i\mu} P_L + (V_R^\dagger)_{\mu i} (V_R)_{i\mu} P_R \right).\tag{A.11}$$

Now we can get the contribution of diagram b to the loop integral:

$$T(b) = e g_2^2 \bar{u}_\mu (i\sigma_{\mu\nu} q^\nu) (m_\mu A F + m_i B G + m_\mu D W) u_\mu,\tag{A.12}$$

where

$$\begin{aligned}
F &= -\frac{1}{4} \frac{\xi}{\xi-1} I_1 + \frac{1}{24} \frac{1}{\xi-1} \left(6 - \frac{r_i}{r-1} \right) I_2 - \frac{1}{24} \frac{r_i}{\xi-1} \left(\frac{1}{\xi-1} + 2 \right) I_3 \\
&\quad + \frac{1}{24} \frac{r_i}{\xi-1} \left(\frac{1}{\xi-1} - \frac{1}{r_i-1} + 2 \right) I_4 + \frac{1}{12} \frac{r_i}{(r_i-1)(\xi-1)} I_7 + \frac{1}{12} r_i m_W^2 \xi I_9 \\
&\quad + \frac{1}{24} r_i m_W^4 \xi^2 I_{10} - \frac{1}{12} r_i m_W^2 \left(1 - \xi + \frac{1}{\xi-1} \right) I_{11} + \frac{1}{12} r_i m_W^4 \xi^2 I_{12}, \\
G &= \frac{1}{4} \frac{\xi}{\xi-1} I_1 - \frac{1}{4} \frac{1}{\xi-1} I_2 + \frac{1}{8} \frac{r_i}{\xi-1} I_3 - \frac{1}{8} \frac{r_i}{\xi-1} I_4 - \frac{1}{8} r_i m_W^2 \xi I_9, \\
W &= \frac{1}{24} \frac{r_i}{(\xi-1)(r_i-1)} I_2 + \frac{r_i}{\xi-1} \left[-\frac{1}{24} + \frac{1}{24(\xi-1)} \right] I_3 \\
&\quad + \frac{r_i}{\xi-1} \left[\frac{1}{24} - \frac{1}{24(\xi-1)} - \frac{1}{24(r_i-1)} \right] I_4 + \frac{1}{24} m_W^2 r_i \xi I_9 - \frac{1}{24} m_W^4 r_i \xi^2 I_{10}.
\end{aligned} \tag{A.13}$$

Note that $r_i = m_i^2/m_W^2$, I_n are integral functions listed in Appendix C. Substituting the I_n functions into $T(b)$ gives:

$$\begin{aligned}
T(b) &= \frac{i}{(4\pi)^2} \frac{1}{m_W^2} e g_2^2 \bar{u}_\mu (i\sigma_{\mu\nu} q^\nu) \left\{ m_\mu A [M(b) + P(b) \ln r_i + Q(b) \ln \xi] \right. \\
&\quad + m_i B [M'(b) + P'(b) \ln r_i + Q'(b) \ln \xi] \\
&\quad \left. + m_\mu D [M_D(b) + P_D(b) \ln r_i + Q_D(b) \ln \xi] \right\} u_\mu,
\end{aligned} \tag{A.14}$$

where $M(b), P(b), Q(b), \dots$ are listed in Appendix E.

Now we proceed to diagram (c) and (d). Following the steps which are entirely similar to those in the calculation of diagram (b), we can obtain the contributions to the amplitude from them:

$$\begin{aligned}
T(c) &= e g_2^2 \bar{u}_\mu (i\sigma_{\mu\nu} q^\nu) \frac{i}{(4\pi)^2} \frac{1}{m_W^2} \left\{ m_\mu A [M(c) + P(c) \ln r_i + Q(c) \ln \xi] \right. \\
&\quad + m_i B [M'(c) + P'(c) \ln r_i + Q'(c) \ln \xi] + m_\mu D [M_D(c) + P_D(c) \ln r_i \\
&\quad \left. + Q_D(c) \ln \xi] \right\} u_\mu,
\end{aligned} \tag{A.15}$$

$$\begin{aligned}
T(d) &= e g_2^2 \bar{u}_\mu (i\sigma_{\mu\nu} q^\nu) \frac{i}{(4\pi)^2} \frac{1}{m_W^2} \left\{ m_\mu A [M(d) + P(d) \ln r_i + Q(d) \ln \xi] \right. \\
&\quad + m_i B [M'(d) + P'(d) \ln r_i + Q'(d) \ln \xi] \\
&\quad \left. + m_\mu D [M_D(d) + P_D(d) \ln r_i + Q_D(d) \ln \xi] \right\} u_\mu,
\end{aligned} \tag{A.16}$$

where $M(c), P(c), M(d), P(d), \dots$ are listed in Appendix F and G.

We finally come to the diagram (a) which is the most complicated one due to the appearance of a triple gauge coupling and double gauge boson propagators:

$$-\frac{2}{e g_2^2} (a) = \bar{u}_\mu \int \frac{d^4 k}{(2\pi)^4} \gamma_\sigma \left[(V_L^\dagger)_{\mu i} P_L + (V_R^\dagger)_{\mu i} P_R \right] \frac{1}{\not{k} + \not{p} - m_i} \gamma_\rho \left[(V_L)_{i\mu} P_L + (V_R)_{i\mu} P_R \right] \Gamma_{\alpha\beta\mu}$$

$$\begin{aligned} & \times \frac{1}{k^2 - m_W^2} \left[g^{\alpha\rho} - \frac{k^\alpha k^\rho}{k^2 - \xi m_W^2} (1 - \xi) \right] \frac{1}{(k+q)^2 - m_W^2} \\ & \times \left[g^{\beta\sigma} - \frac{(k+q)^\beta (k+q)^\sigma}{(k+q)^2 - \xi m_W^2} (1 - \xi) \right] u_\mu. \end{aligned} \quad (\text{A.17})$$

The above can be split into three terms corresponding to the product of the two propagators:

$$-\frac{2}{eg_2^2}(a) = (gg) + (k) + (k+q) \quad (\text{A.18})$$

where

$$\begin{aligned} (gg) &= \bar{u}_\mu \int \frac{d^4k}{(2\pi)^4} \left[(V_L^\dagger)_{\mu i} P_R + (V_R^\dagger)_{\mu i} P_L \right] \gamma^\beta \frac{1}{\not{k} + \not{p} - m_i} \gamma^\alpha \left[(V_L)_{i\mu} P_L + (V_R)_{i\mu} P_R \right] \\ & \times \frac{\Gamma_{\alpha\beta\mu}}{[k^2 - m_W^2][(k+q)^2 - m_W^2]} u_\mu, \end{aligned} \quad (\text{A.19})$$

$$\begin{aligned} -\frac{(k)}{1-\xi} &= \bar{u}_\mu \int \frac{d^4k}{(2\pi)^4} \gamma^\beta \left[(V_L^\dagger)_{ei} P_L + (V_R^\dagger)_{\mu i} P_R \right] \frac{1}{\not{k} + \not{p} - m_i} \not{k} \left[(V_L)_{i\mu} P_L + (V_R)_{i\mu} P_R \right] \\ & \times \frac{k^\alpha \Gamma_{\alpha\beta\mu}}{[k^2 - m_W^2][k^2 - \xi m_W^2][(k+q)^2 - m_W^2]} u_\mu, \end{aligned} \quad (\text{A.20})$$

$$\begin{aligned} -\frac{(k+q)}{1-\xi} &= \bar{u}_\mu \int \frac{d^4k}{(2\pi)^4} (\not{k} + \not{q}) \left[(V_L^\dagger)_{\mu i} P_L + (V_R^\dagger)_{\mu i} P_R \right] \frac{1}{\not{k} + \not{p} - m_i} \gamma^\alpha \left[(V_L)_{i\mu} P_L + (V_R)_{i\mu} P_R \right] \\ & \times \frac{(k+q)^\beta \Gamma_{\alpha\beta\mu}}{[k^2 - m_W^2][(k+q)^2 - m_W^2][(k+q)^2 - \xi m_W^2]} u_\mu. \end{aligned} \quad (\text{A.21})$$

The fourth term from the product of propagators has been discarded since it does not contribute to the on-shell amplitude.

We start from the apparently easiest (actually the most complicated) term (gg) . Simplify by aiming at the leading terms linear in m_μ :

$$\gamma^\beta \frac{1}{\not{k} + \not{p} - m_i} \gamma^\alpha \Gamma_{\alpha\beta\mu}. \quad (\text{A.22})$$

Note that the term q_μ and the term γ_μ can be dropped because they have no contribution to (gg) and we use $\gamma_\alpha (\not{k} + \not{p}) \gamma^\alpha = -2(\not{k} + \not{p})$. Thus we get:

$$\begin{aligned} & \gamma^\beta \frac{1}{\not{k} + \not{p} - m_i} \gamma^\alpha \Gamma_{\alpha\beta\mu} \\ & \rightarrow \frac{1}{(k+p)^2 - m_i^2} \left[4\not{k}k_\mu + 2k_\mu(2\not{q} - 3m_i) + \gamma_\mu \not{k}(2\not{q} - \not{p}) + \gamma_\mu 2\not{q}(-\not{p} + m_i) + m_i(-\not{q})\gamma_\mu \right. \\ & \quad \left. - 3m_\mu \not{q}\gamma_\mu + 4m_\mu k_\mu - m_\mu \not{k}\gamma_\mu \right]. \end{aligned} \quad (\text{A.23})$$

Substitute (A.23) into (A.19) and again we should pick out $(i\sigma_{\mu\nu}q^\nu)$ terms. By using the same method as in calculating $(b)_I$ we get the following contribution to the loop integral:

$$T(gg) = \bar{u}_\mu (i\sigma_{\mu\nu}q^\nu) \left[m_\mu AM + m_i BN + m_\mu BH + m_\mu DS \right] u_\mu, \quad (\text{A.24})$$

where

$$M = \frac{4 - 13r_i + 9r_i^2}{6(r_i - 1)^2} I_2 + \frac{(-5r_i + 2)r_i}{3(r_i - 1)^2} I_4 + \frac{2}{3} \frac{r_i^2}{(r_i - 1)^2} I_7 + \frac{3r_i - 1}{6(r_i - 1)^2} I_8,$$

$$\begin{aligned}
N &= \frac{3(-2r_i+1)}{2(r_i-1)}I_2 + \frac{3r_i}{2(r_i-1)}I_4, \\
H &= -3I_2 + \frac{1}{2}\frac{r_i}{r_i-1}I_4 - \frac{1}{2}\frac{1}{r_i-1}I_8, \\
S &= \frac{2r_i}{3(r_i-1)^2}I_2 - \frac{r_i^2}{3(r_i-1)^2}I_4 - \frac{1}{3(r_i-1)^2}I_8.
\end{aligned} \tag{A.25}$$

For (A.20), note that:

$$k_\alpha \Gamma_{\alpha\beta\mu} \rightarrow -k_\mu(k+q)_\beta + g\beta_\mu(2q+k) \cdot k, \tag{A.26}$$

we simplify the structure:

$$\begin{aligned}
&\gamma^\beta \left(V_L^\dagger P_L + V_R^\dagger P_R \right) \frac{k^\alpha \Gamma_{\alpha\beta\mu}}{\not{k} + \not{p} - m_i} \not{k} \\
&\rightarrow \text{I}'' + \text{II}'' + \text{III}''.
\end{aligned} \tag{A.27}$$

where

$$\begin{aligned}
\text{I}'' &= - \left(V_L^\dagger P_R + V_R^\dagger P_L \right) m_i \frac{k_\mu (\not{k} + \not{p} + m_i)(m_i - \not{p})}{(k+p)^2 - m_i^2}, \\
\text{II}'' &= \left(V_L^\dagger P_L + V_R^\dagger P_R \right) m_\mu \frac{k_\mu (\not{k} + \not{p} + m_i)(m_i - \not{p})}{(k+p)^2 - m_i^2}, \\
\text{III}'' &= \left(V_L^\dagger P_R + V_R^\dagger P_L \right) \gamma_\mu (2k \cdot q + k^2) \frac{(\not{k} + \not{p} + m_i)(m_i - \not{p})}{(k+p)^2 - m_i^2}.
\end{aligned} \tag{A.28}$$

Using the same method in diagram b we get the contribution of the above three terms:

$$\begin{aligned}
T_{\text{I}'} &= (i\sigma_{\mu\nu}q^\nu) \int_k \left\{ m_i^2 m_\mu A \left[-\frac{1}{12} \frac{k^4}{(k^2 - m_i^2)^2 (k^2 - \xi m_W^2)(k^2 - m_W^2)^3} \right. \right. \\
&\quad \left. \left. - \frac{1}{6} \frac{k^4}{(k^2 - m_i^2)^3 (k^2 - \xi m_W^2)(k^2 - m_W^2)^2} \right] - \frac{1}{4} m_i^3 B \frac{k^2}{(k^2 - m_i^2)^2 (k^2 - \xi m_W^2)(k^2 - m_W^2)^2} \right. \\
&\quad \left. + \frac{1}{12} m_i^2 m_\mu D \frac{k^4}{(k^2 - m_i^2)^2 (k^2 - \xi m_W^2)(k^2 - m_W^2)^3} \right\}. \\
T_{\text{II}'} &= (i\sigma_{\mu\nu}q^\nu) m_i^2 m_\mu D \left(-\frac{1}{4} \right) \int_k \frac{k^2}{(k^2 - m_i^2)^2 (k^2 - \xi m_W^2)(k^2 - m_W^2)^2}. \\
T_{\text{III}'} &= (i\sigma_{\mu\nu}q^\nu) \left\{ m_\mu A \left[-\frac{1}{2} \frac{k^4}{(k^2 - m_i^2)(k^2 - \xi m_W^2)(k^2 - m_W^2)^3} \right. \right. \\
&\quad \left. \left. + \frac{1}{2} \frac{k^2}{(k^2 - m_i^2)(k^2 - \xi m_W^2)(k^2 - m_W^2)^2} \right] + m_i B \left[\frac{1}{2} \frac{k^4}{(k^2 - m_i^2)(k^2 - \xi m_W^2)(k^2 - m_W^2)^3} \right. \right. \\
&\quad \left. \left. - \frac{1}{2} \frac{k^2}{(k^2 - m_i^2)(k^2 - \xi m_W^2)(k^2 - m_W^2)^2} \right] \right\}.
\end{aligned} \tag{A.29}$$

Thus we can get:

$$T \left[-\frac{(k)}{1-\xi} \right] = \bar{u}_\mu (i\sigma_{\mu\nu}q^\nu) \left[m_\mu A M' + m_i B N' + m_\mu D W' \right] u_\mu, \tag{A.30}$$

where

$$\begin{aligned}
M' &= \frac{6 + r_i - 12\xi + 4r_i\xi}{(r_i - 1)(1 - \xi)^2} I_2 + \frac{r_i \xi^2}{12(1 - \xi)^3} I_3 + \frac{r_i(2 + r_i - 3\xi - 3r_i\xi + 4r_i\xi^2 - r_i^2\xi^2)}{12(r_i - 1)^2(1 - \xi)^3} I_4 \\
&\quad - \frac{r_i(r_i + \xi - 2r_i\xi)}{6(r_i - 1)^2(1 - \xi)^2} I_7 + \frac{5r_i - 6}{12(r_i - 1)^2(1 - \xi)} I_8 - \frac{r_i \xi^2 m_W^2}{6(1 - \xi)^2} I_{11}, \\
N' &= \frac{-2 - r_i + 4\xi - r_i\xi}{4(r_i - 1)(1 - \xi)^2} I_2 + \frac{r_i(\xi - 2)}{4(1 - \xi)^2} I_3 + \frac{r_i(1 - r_i\xi)}{4(r_i - 1)(1 - \xi)^2} I_4 - \frac{1}{2(r_i - 1)(1 - \xi)} I_8, \\
W' &= \frac{-r_i(4 - 2r_i + r_i\xi - 3\xi)}{12(r_i - 1)^2(\xi - 1)^2} I_2 + \frac{r_i\xi(3 - 2\xi)}{12(\xi - 1)^3} I_3 + \left[\frac{r_i(r_i\xi - 1)}{4(r_i - 1)(\xi - 1)^2} \right. \\
&\quad \left. - \frac{r_i(r_i + \xi - 3r_i\xi + r_i^2\xi^2)}{12(r_i - 1)^2(\xi - 1)^3} \right] I_4 - \frac{r_i}{12(r_i - 1)^2(\xi - 1)} I_8.
\end{aligned} \tag{A.31}$$

Now we cope with the last term (A.21). Simplify first the combination:

$$(\not{k} + \not{q}) \left(V_L^\dagger P_L + V_R^\dagger P_R \right) \frac{(k + q)^\beta \Gamma_{\alpha\beta\mu}}{\not{k} + \not{p} - m_i} \gamma^\alpha \rightarrow \text{I}(m_i) + \text{II}(m_\mu). \tag{A.32}$$

where

$$\begin{aligned}
\text{I}(m_i) &= \left(V_L^\dagger P_R + V_R^\dagger P_L \right) m_i \frac{-m_i k_\mu \not{k} + k_\mu \not{k} \not{p} - m_i^2 k_\mu + k^2 \not{k} \gamma_\mu + k^2 \not{p} \gamma_\mu + m_i k^2 \gamma_\mu}{(k + p)^2 - m_i^2}, \\
\text{II}(m_\mu) &= - \left(V_L^\dagger P_L + V_R^\dagger P_R \right) m_\mu \frac{1}{\not{k} + \not{p} - m_i} (-k_\mu \not{k} + \gamma_\mu k^2).
\end{aligned} \tag{A.33}$$

We can deal with the above two terms using the same method in diagram b. Thus we get the contribution of them to the loop integral:

$$\begin{aligned}
T[\text{I}(m_i)] &= (i\sigma_{\mu\nu} q^\nu) \int_k \left\{ m_i^2 m_\mu A \left[-\frac{1}{12} \frac{k^4}{(k^2 - m_i^2)^2 (k^2 - \xi m_W^2)^2 (k^2 - m_W^2)^2} \right. \right. \\
&\quad \left. - \frac{1}{12} \frac{k^4}{(k^2 - m_i^2)^2 (k^2 - \xi m_W^2) (k^2 - m_W^2)^3} - \frac{1}{6} \frac{k^4}{(k^2 - m_i^2)^3 (k^2 - \xi m_W^2) (k^2 - m_W^2)^2} \right] \\
&\quad + m_i^3 B \frac{1}{4} \frac{k^2}{(k^2 - m_i^2)^2 (k^2 - \xi m_W^2) (k^2 - m_W^2)^2} \\
&\quad + m_i B \left[-\frac{1}{2} \frac{k^4}{(k^2 - m_i^2)^2 (k^2 - \xi m_W^2) (k^2 - m_W^2)^2} \right. \\
&\quad \left. - \frac{1}{2} \frac{k^4}{(k^2 - m_i^2) (k^2 - \xi m_W^2)^2 (k^2 - m_W^2)^2} - \frac{1}{2} \frac{k^4}{(k^2 - m_i^2) (k^2 - \xi m_W^2) (k^2 - m_W^2)^3} \right. \\
&\quad \left. + \frac{k^2}{(k^2 - m_i^2) (k^2 - \xi m_W^2) (k^2 - m_W^2)^2} \right] \\
&\quad \left. + m_i^2 m_\mu D \left[\frac{1}{12} \frac{k^4}{(k^2 - m_i^2)^2 (k^2 - \xi m_W^2)^2 (k^2 - m_W^2)^2} \right. \right. \\
&\quad \left. + \frac{1}{12} \frac{k^4}{(k^2 - m_i^2)^2 (k^2 - \xi m_W^2) (k^2 - m_W^2)^3} \right] \left. \right\}.
\end{aligned} \tag{A.34}$$

$$T[\text{II}(m_\mu)] \rightarrow (i\sigma_{\mu\nu} q^\nu) m_\mu D \left[-\frac{1}{4} m_i^2 \frac{k^2}{(k^2 - m_i^2)^2 (k^2 - \xi m_W^2) (k^2 - m_W^2)^2} \right]$$

$$\begin{aligned}
& + \frac{1}{2} \frac{k^4}{(k^2 - m_i^2)^2 (k^2 - \xi m_W^2) (k^2 - m_W^2)^2} + \frac{1}{2} \frac{k^4}{(k^2 - m_i^2) (k^2 - \xi m_W^2)^2 (k^2 - m_W^2)^2} \\
& + \frac{1}{2} \frac{k^4}{(k^2 - m_i^2) (k^2 - \xi m_W^2) (k^2 - m_W^2)^3} - \frac{k^2}{(k^2 - m_i^2) (k^2 - \xi m_W^2) (k^2 - m_W^2)^2} \Big].
\end{aligned} \tag{A.35}$$

At last we obtain the following contribution to the loop integral:

$$T \left[-\frac{(k+q)}{1-\xi} \right] = \bar{u}_\mu (i\sigma_{\mu\nu} q^\nu) \left[m_\mu A M'' + m_i B N'' + m_\mu D W'' \right] u_\mu, \tag{A.36}$$

where

$$\begin{aligned}
M'' &= \frac{r_i}{6(r_i-1)(1-\xi)} I_2 + \frac{r_i \xi (\xi-2)}{12(1-\xi)^3} I_3 \\
&+ \frac{r_i(3-2\xi-6r_i\xi+2r_i^2\xi+4r_i\xi^2-r_i^2\xi^2)}{12(1-\xi)^3(r_i-1)^2} I_4 + \frac{r_i(-r_i-\xi+2r_i\xi)}{6(r_i-1)^2(1-\xi)^2} I_7 \\
&- \frac{r_i}{12(r_i-1)^2(1-\xi)} I_8 - \frac{r_i \xi^2 m_W^2}{12(1-\xi)^2} I_9 - \frac{r_i \xi^2 m_W^2}{6(1-\xi)^2} I_{11}, \\
N'' &= \frac{1}{2(1-\xi)^2} I_1 - \frac{r_i}{4(r_i-1)(1-\xi)} I_2 - \frac{\xi^2}{2(1-\xi)^2} I_3 \\
&+ \frac{-r_i-2\xi+4r_i\xi-r_i^2\xi}{4(r_i-1)(1-\xi)^2} I_4 - \frac{\xi^2}{2(1-\xi)^2} I_5 + \frac{1}{2(r_i-1)(1-\xi)} I_8, \\
W'' &= -\frac{\xi^2}{2(\xi-1)^2} I_1 + \left[\frac{r_i^2}{6(\xi-1)(r_i-1)^2} - \frac{r_i}{4(r_i-1)(\xi-1)} \right] I_2 \\
&+ \left[\frac{r_i \xi (\xi-2)}{12(\xi-1)^3} + \frac{\xi(-r_i+2\xi)}{4(\xi-1)^2} \right] I_3 \\
&+ \left[\frac{r_i-2r_i^2+2r_i^3\xi-r_i^3\xi^2}{12(r_i-1)^2(\xi-1)^3} + \frac{r_i+2\xi-4r_i\xi+r_i^2\xi}{4(r_i-1)(\xi-1)^2} \right] I_4 + \frac{\xi^2}{2(\xi-1)^2} I_5 \\
&+ \left[\frac{1}{2(r_i-1)(\xi-1)} - \frac{r_i}{12(\xi-1)(r_i-1)^2} \right] I_8 + \frac{1}{12} m_W^2 \frac{r_i \xi^2}{(\xi-1)^2} I_9.
\end{aligned} \tag{A.37}$$

Using (A.24), (A.30) and (A.36), we finally obtain the contribution from diagram (a):

$$\begin{aligned}
T(a) &= eg_2^2 \bar{u}_\mu (i\sigma_{\mu\nu} q^\nu) \frac{i}{(4\pi)^2} \frac{1}{m_W^2} \left\{ m_\mu A \left[M(a) + P(a) \ln[r_i] + Q(a) \ln[\xi] \right] \right. \\
&+ m_i B \left[M'(a) + P'(a) \ln[r_i] + Q'(a) \ln[\xi] \right] \\
&+ m_\mu B \left[M''(a) + P''(a) \ln[r_i] \right] \\
&\left. + m_\mu D \left[M_D(a) + P_D(a) \ln[r_i] + Q_D(a) \ln[\xi] \right] \right\} u_\mu,
\end{aligned} \tag{A.38}$$

where $M(a), P(a), Q(a), \dots$ are listed in Appendix H.

Appendix B. The calculation of the contribution from Z diagram

We start with diagram (b): We begin to calculate the amplitude of diagram Z whose Feynman diagrams are shown in Fig 2.

$$(b)_Z = -\frac{eg_2^2}{4c_W} \bar{u}_\mu \int_k \frac{(c_{\mu\alpha}^L P_R + c_{\mu\alpha}^R P_L) N_\mu (c_{\alpha\mu}^L P_L + c_{\alpha\mu}^R P_R)}{(k-p)^2 - m_Z^2} u_\mu, \quad (\text{B.1})$$

$$\begin{aligned} N_\mu &= \gamma_\sigma \frac{1}{\not{k} - \not{q} - m_\alpha} \gamma_\mu \frac{1}{\not{k} - m_\alpha} \gamma_\rho \left(g^{\rho\sigma} - (1-\xi) \frac{(k-p)^\rho (k-p)^\sigma}{(k-p)^2 - \xi m_Z^2} \right) \\ &= \text{1st} + \text{2nd}, \\ c_{\alpha\beta}^L &= (V_L^\dagger V_L)_{\alpha\beta} - 2s_W^2 \delta_{\alpha\beta}, c_{\alpha\beta}^R = (V_R^\dagger V_R)_{\alpha\beta} - 2s_W^2 \delta_{\alpha\beta} \end{aligned} \quad (\text{B.2})$$

where

$$\begin{aligned} \text{1st} &= \gamma^\rho \frac{1}{\not{k} - \not{q} - m_\alpha} \gamma_\mu \frac{1}{\not{k} - m_\alpha} \gamma_\rho, \\ \text{2nd} &= -(1-\xi)(\not{k} - \not{p}) \frac{1}{\not{k} - \not{q} - m_\alpha} \gamma_\mu \frac{1}{\not{k} - m_\alpha} (\not{k} - \not{p}) \frac{1}{(k-p)^2 - \xi m_Z^2}. \end{aligned} \quad (\text{B.3})$$

Simplify the term 1st. Note that the terms γ_μ, q_μ can be dropped because they have no contribution to the loop integral. Thus we can get:

$$\text{1st} \rightarrow \frac{-4k_\mu \not{k} + 2\not{k} \gamma_\mu \not{q} + 8m_\alpha k_\mu}{[(k-q)^2 - m_\alpha^2][k^2 - m_\alpha^2]}. \quad (\text{B.4})$$

Substituting (B.4) into (B.1) we can get the contribution of the term 1st to the loop integral:

$$T(\text{1st}) = \frac{eg_2^2}{4c_W} \bar{u}_\mu (i\sigma_{\mu\nu} q^\nu) [m_\mu A'(FF) + m_\alpha B'(GG) + m_\mu D'(WW)] u_\mu. \quad (\text{B.5})$$

where

$$\begin{aligned} (FF) &= \frac{r_\alpha^2}{3(r_\alpha - 1)^2} T_7 + \frac{2r_\alpha}{3(r_\alpha - 1)} T_4 + \frac{1 - 4r_\alpha}{3(r_\alpha - 1)^2} T_2 + \frac{2}{3(r_\alpha - 1)^2} T_8, \\ (GG) &= \frac{2r_\alpha}{r_\alpha - 1} T_4 - \frac{2}{r_\alpha - 1} T_2, \\ (WW) &= -\frac{r_\alpha^2}{3(r_\alpha - 1)^2} T_7 + \frac{r_\alpha(5 - 3r_\alpha)}{3(r_\alpha - 1)^2} T_4 + \frac{3r_\alpha - 4}{3(r_\alpha - 1)^2} T_2. \end{aligned} \quad (\text{B.6})$$

where $r_\alpha = m_\alpha^2/m_W^2$, and T_n are integral functions listed in Appendix.

Now we proceed to the term 2nd. To pick up the relevant terms, again we will use:

$$\begin{aligned} k_\mu \not{k} (k \cdot p) (k \cdot q) &\rightarrow \frac{1}{48} (i\sigma_{\mu\nu} q^\nu) \not{p} k^4 - \frac{1}{48} (i\sigma_{\mu\nu} q^\nu) m_\mu^* k^4, \\ k_\mu \not{k} (k \cdot p) (k \cdot p) &\rightarrow \frac{1}{24} (i\sigma_{\mu\nu} q^\nu) \not{p} k^4. \end{aligned} \quad (\text{B.7})$$

We can also use the replacement $\not{p} \rightarrow m_\mu$ when \not{p} is on the far right. Thus we can get the contribution of the term 2nd to the loop integral:

$$T(\text{2nd}) = \frac{eg_2^2}{4c_W} \bar{u}_\mu (i\sigma_{\mu\nu} q^\nu) [m_\mu A'(FF)' + m_\alpha B'(GG)' + m_\mu D'(WW)'] u_\mu \quad (\text{B.8})$$

where

$$\begin{aligned}
(FF)' &= \left[-\frac{1}{2} \frac{r_\alpha}{(r_\alpha - 1)^2} - \frac{5}{3} \frac{r_\alpha}{r_\alpha - 1} \right] T_2 - \frac{7}{3} r_\alpha T_3 + \left[\frac{4}{3} \frac{r_\alpha}{r_\alpha - 1} + \frac{7}{3} r_\alpha \right] T_4 \\
&\quad + \left[\frac{1}{3} \frac{r_\alpha}{r_\alpha - 1} + \frac{1}{6} \frac{r_\alpha}{(r_\alpha - 1)^2} \right] T_7 + \frac{1}{3} \frac{r_\alpha}{(r_\alpha - 1)^2} T_8 - \frac{5}{3} m_Z^2 r_\alpha \xi T_9 - \frac{1}{3} m_Z^4 r_\alpha \xi^2 T_{10} \\
&\quad - \frac{1}{3} m_Z^2 r_\alpha \xi T_{11} - \frac{1}{6} m_Z^4 r_\alpha \xi^2 T_{12}, \\
(GG)' &= \frac{3}{2} (\xi - 1) T_1 + \frac{1}{2(r_\alpha - 1)} T_2 + \frac{\xi + 3r_\alpha}{2} T_3 + \left[-\frac{1}{2} - \frac{3r_\alpha}{2} - \frac{r_\alpha}{2(s_i - 1)} \right] T_4 \\
&\quad + \frac{1}{2} \xi T_5 + \frac{1}{2} m_Z^2 r_\alpha \xi T_9, \\
(WW)' &= -\frac{3}{2} (\xi - 1) T_1 + \frac{-4r_\alpha + 3}{6(r_\alpha - 1)^2} T_2 - \frac{\xi + 3r_\alpha}{2} T_3 + \left[\frac{r_\alpha^2}{3(r_\alpha - 1)^2} + \frac{1}{2} + \frac{3r_\alpha}{2} \right. \\
&\quad \left. + \frac{r_\alpha}{2(r_\alpha - 1)} \right] T_4 - \frac{1}{2} \xi T_5 + \frac{-2r_\alpha^2 + r_\alpha}{6(r_\alpha - 1)^2} T_7 - \frac{1}{2} m_Z^2 r_\alpha \xi T_9 + \frac{1}{3} m_Z^2 r_\alpha \xi T_{11} \\
&\quad + \frac{1}{6} m_Z^4 r_\alpha \xi^2 T_{12}.
\end{aligned}$$

$$\begin{aligned}
A' &= \sum_\alpha (c_{\alpha\mu}^L c_{\alpha\mu}^L P_R + c_{\alpha\mu}^R c_{\alpha\mu}^R P_L), \\
B' &= \sum_i (c_{\alpha\mu}^L c_{\alpha\mu}^R P_R + c_{\alpha\mu}^R c_{\alpha\mu}^L P_L), \\
D' &= \sum_\alpha (c_{\alpha\mu}^L c_{\alpha\mu}^L P_L + c_{\alpha\mu}^R c_{\alpha\mu}^R P_R).
\end{aligned} \tag{B.9}$$

At last we get the contribution of the diagram $(b)_Z$

$$\begin{aligned}
T[(b)_Z] &= \frac{eg_2^2}{4c_W m_Z^2} \frac{i}{(4\pi)^2} \bar{u}_\mu (i\sigma_{\mu\nu} q^\nu) \left\{ m_\mu A' [MM(b) + PP(b) \ln r_\alpha + QQ(b) \ln \xi] \right. \\
&\quad + m_\alpha B' [MM'(b) + PP'(b) \ln r_\alpha + QQ'(b) \ln \xi] \\
&\quad \left. + m_\mu D' [MM_D(b) + PP_D(b) \ln r_\alpha + QQ_D(b) \ln \xi] \right\} u_\mu,
\end{aligned} \tag{B.10}$$

where $MM(b), PP(b), QQ(b), \dots$ are listed in Appendix I.

Now replacing Z by G^0 , we get:

$$(b)_{G^0} = -\frac{eg_2^2}{4c_W m_Z^2} \bar{u}_\mu \int_k \frac{H}{(k-p)^2 - \xi m_Z^2} u_\mu. \tag{B.11}$$

where

$$\begin{aligned}
H &= \left[m_\alpha (c_{\mu\alpha}^L P_R + c_{\mu\alpha}^R P_L) - m_\mu (c_{\mu\alpha}^L P_L + c_{\mu\alpha}^R P_R) \right] N \left[m_\mu (c_{\alpha\mu}^L P_R + c_{\alpha\mu}^R P_L) \right. \\
&\quad \left. - m_\alpha (c_{\alpha\mu}^L P_L + c_{\alpha\mu}^R P_R) \right], \\
N &= \frac{1}{\not{k} - \not{q} - m_\alpha} \gamma_\mu \frac{1}{\not{k} - m_\alpha}.
\end{aligned} \tag{B.12}$$

Simplify N we get:

$$N \rightarrow \frac{2k_\mu \not{k} + m_\alpha \not{k} \gamma_\mu - \not{q} \gamma_\mu \not{k} - m_\alpha \not{q} \gamma_\mu + m_\alpha \gamma_\mu \not{k}}{[(k-q)^2 - m_\alpha^2][k^2 - m_\alpha^2]} \tag{B.13}$$

Substituting (B.13) into (B.11) and using the same method in the diagram b, we can get the contribution of the b_G^0 to the amplitude:

$$\begin{aligned}
 T[b_{G^0}] = & \frac{eg_2^2}{4c_W m_Z^2} \frac{i}{(4\pi)^2} \bar{u}_\mu (i\sigma_{\mu\nu} q^\nu) \left\{ m_\mu A' [MM(c) + PP(c) \ln r_\alpha + QQ(c) \ln \xi] \right. \\
 & + m_\alpha B' [MM'(c) + PP'(c) \ln r_\alpha + QQ'(c) \ln \xi] \\
 & \left. + m_\mu D' [MM_D(c) + PP_D(c) \ln r_\alpha + QQ_D(c) \ln \xi] \right\} u_\mu, \tag{B.14}
 \end{aligned}$$

where $MM(c), PP(c), QQ(c), \dots$ are listed in Appendix I.

Appendix C. The loop functions appearing in Fig 1 are defined and the results are listed

$$\begin{aligned}
I_1 &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_i^2)(k^2 - \xi m_W^2)(k^2 - m_W^2)} = \frac{i}{(4\pi)^2} \frac{(\xi - 1)r_i \ln r_i - (r_i - 1)\xi \ln \xi}{m_W^2(1 - \xi)(r_i - 1)(r_i - \xi)}, \\
I_2 &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_i^2)(k^2 - m_W^2)^2} = -\frac{i}{(4\pi)^2} \frac{1}{m_W^2(1 - r_i)} \left(1 + \frac{r_i \ln r_i}{1 - r_i}\right), \\
I_3 &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_i^2)^2(k^2 - \xi m_W^2)} = -\frac{i}{(4\pi)^2} \frac{1}{m_W^2(r_i - \xi)} \left(1 - \frac{\xi \ln \frac{r_i}{\xi}}{r_i - \xi}\right), \\
I_4 &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_i^2)^2(k^2 - m_W^2)} = -\frac{i}{(4\pi)^2} \frac{1}{m_W^2(r_i - 1)} \left(1 - \frac{\ln r_i}{r_i - 1}\right), \\
I_5 &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_i^2)(k^2 - \xi m_W^2)^2} = \frac{i}{(4\pi)^2} \frac{1}{m_W^2(r_i - \xi)} \left(1 + \frac{r_i \ln \frac{\xi}{r_i}}{r_i - \xi}\right), \\
I_6 &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - \xi m_W^2)^3} = -\frac{i}{(4\pi)^2} \frac{1}{2} \frac{1}{\xi m_W^2}, \\
I_7 &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_i^2)^3} = -\frac{i}{(4\pi)^2} \frac{1}{2} \frac{1}{r_i m_W^2}, \\
I_8 &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_W^2)^3} = -\frac{i}{(4\pi)^2} \frac{1}{2} \frac{1}{m_W^2}, \\
I_9 &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_i^2)^2(k^2 - \xi m_W^2)^2} = -\frac{i}{(4\pi)^2} \frac{1}{m_W^4(r_i - \xi)^2} \left(2 + \frac{(r_i + \xi) \ln \frac{\xi}{r_i}}{(r_i - \xi)}\right), \\
I_{10} &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_i^2)^2(k^2 - \xi m_W^2)^3} = \frac{i}{(4\pi)^2 m_W^6(r_i - \xi)^3} \left(-\frac{r_i + 5\xi}{2\xi} - \frac{2r_i + \xi}{r_i - \xi} \ln \frac{\xi}{r_i}\right), \\
I_{11} &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_i^2)^3(k^2 - \xi m_W^2)} = \frac{i}{(4\pi)^2 m_W^4(r_i - \xi)^2} \left(\frac{1}{2} + \frac{\xi}{2r_i} - \frac{\xi \ln \frac{r_i}{\xi}}{r_i - \xi}\right), \\
I_{12} &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_i^2)^3(k^2 - \xi m_W^2)^2} = \frac{i}{(4\pi)^2 m_W^6(r_i - \xi)^3} \left(\frac{5r_i + \xi}{2r_i} + \frac{r_i + 2\xi}{r_i - \xi} \ln \frac{r_i}{\xi}\right), \\
I_{13} &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_i^2)(k^2 - \xi m_W^2)^3} = \frac{i}{(4\pi)^2} \frac{1}{2m_W^4(r_i - \xi)^2} \left(1 + \frac{r_i}{\xi} + \frac{2r_i \ln \frac{\xi}{r_i}}{r_i - \xi}\right).
\end{aligned}$$

Appendix D. The loop functions appearing in Fig 2 are defined and the results are listed.

$$\begin{aligned}
T_1 &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_\alpha^2)(k^2 - \xi m_Z^2)(k^2 - m_Z^2)} = \frac{i}{(4\pi)^2} \frac{(\xi - 1)r_\alpha \ln r_\alpha - (r_\alpha - 1)\xi \ln \xi}{m_Z^2(1 - \xi)(r_\alpha - 1)(r_\alpha - \xi)}, \\
T_2 &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_\alpha^2)(k^2 - m_Z^2)^2} = -\frac{i}{(4\pi)^2} \frac{1}{m_Z^2(1 - r_\alpha)} \left(1 + \frac{r_\alpha \ln r_\alpha}{1 - r_\alpha}\right), \\
T_3 &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_\alpha^2)^2(k^2 - \xi m_Z^2)} = -\frac{i}{(4\pi)^2} \frac{1}{m_Z^2(r_\alpha - \xi)} \left(1 - \frac{\xi \ln \frac{r_\alpha}{\xi}}{r_\alpha - \xi}\right), \\
T_4 &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_\alpha^2)^2(k^2 - m_Z^2)} = -\frac{i}{(4\pi)^2} \frac{1}{m_Z^2(r_\alpha - 1)} \left(1 - \frac{\ln r_\alpha}{r_\alpha - 1}\right), \\
T_5 &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_\alpha^2)(k^2 - \xi m_Z^2)^2} = \frac{i}{(4\pi)^2} \frac{1}{m_Z^2(r_\alpha - \xi)} \left(1 + \frac{r_\alpha \ln \frac{\xi}{r_\alpha}}{r_\alpha - \xi}\right), \\
T_6 &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - \xi m_Z^2)^3} = -\frac{i}{(4\pi)^2} \frac{1}{2} \frac{1}{\xi m_Z^2}, \\
T_7 &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_\alpha^2)^3} = -\frac{i}{(4\pi)^2} \frac{1}{2} \frac{1}{r_\alpha m_Z^2}, \\
T_8 &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_Z^2)^3} = -\frac{i}{(4\pi)^2} \frac{1}{2} \frac{1}{m_Z^2}, \\
T_9 &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_\alpha^2)^2(k^2 - \xi m_Z^2)^2} = -\frac{i}{(4\pi)^2} \frac{1}{m_Z^4(r_\alpha - \xi)^2} \left(2 + \frac{(r_\alpha + \xi) \ln \frac{\xi}{r_\alpha}}{(r_\alpha - \xi)}\right), \\
T_{10} &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_\alpha^2)^2(k^2 - \xi m_Z^2)^3} = \frac{i}{(4\pi)^2 m_Z^6(r_\alpha - \xi)^3} \left(-\frac{r_\alpha + 5\xi}{2\xi} - \frac{2r_\alpha + \xi}{r_\alpha - \xi} \ln \frac{\xi}{r_\alpha}\right), \\
T_{11} &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_\alpha^2)^3(k^2 - \xi m_Z^2)} = \frac{i}{(4\pi)^2 m_Z^4(r_\alpha - \xi)^2} \left(\frac{1}{2} + \frac{\xi}{2r_\alpha} - \frac{\xi \ln \frac{r_\alpha}{\xi}}{r_\alpha - \xi}\right), \\
T_{12} &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_\alpha^2)^3(k^2 - \xi m_Z^2)^2} = \frac{i}{(4\pi)^2 m_Z^6(r_\alpha - \xi)^3} \left(\frac{5r_\alpha + \xi}{2r_\alpha} + \frac{r_\alpha + 2\xi}{r_\alpha - \xi} \ln \frac{r_\alpha}{\xi}\right), \\
T_{13} &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_\alpha^2)(k^2 - \xi m_Z^2)^3} = \frac{i}{(4\pi)^2} \frac{1}{2m_Z^4(r_\alpha - \xi)^2} \left(1 + \frac{r_\alpha}{\xi} + \frac{2r_\alpha \ln \frac{\xi}{r_\alpha}}{r_\alpha - \xi}\right).
\end{aligned}$$

Appendix E. The loop functions appearing in diagram(b) are listed.

$$\begin{aligned}
M(b) &= \frac{-5 + r_i + 5\xi - 2r_i\xi}{24(\xi - 1)^2(r_i - 1)} + \frac{r_i - r_i^2 + \xi + 2r_i^2\xi - \xi^2 - 2r_i\xi^2}{24(\xi - 1)^2(r_i - \xi)^2} \\
&\quad + \frac{-2r_i^2 - 7r_i^2\xi + 2\xi^2 + 13r_i\xi^2}{48(r_i - \xi)^3}, \\
P(b) &= r_i \left[\frac{\xi}{4(\xi - 1)(r_i - 1)(r_i - \xi)} - \frac{-4 + 3\xi}{24(\xi - 1)^2(r_i - 1)^2} \right. \\
&\quad \left. + \frac{\xi(-2 + r_i + \xi - 2r_i\xi + 2\xi^2)}{24(r_i - \xi)^3(\xi - 1)^2} - \frac{\xi^3}{8(r_i - \xi)^4} \right], \\
Q(b) &= \frac{\xi}{(r_i - \xi)^2} \left[\frac{-r_i - 4r_i\xi + 6\xi^2}{24(\xi - 1)^2} + \frac{r_i(r_i - \xi - r_i\xi)}{12(\xi - 1)(r_i - \xi)} + \frac{r_i\xi^2}{8(r_i - \xi)^2} \right], \\
M'(b) &= \frac{1}{r_i - \xi} \left[\frac{-r_i + 2\xi - r_i\xi}{8(\xi - 1)(r_i - 1)} + \frac{r_i\xi}{4(r_i - \xi)} \right], \\
P'(b) &= \frac{r_i}{(r_i - \xi)^2} \left[\frac{r_i^2 + \xi - 2r_i\xi - r_i^2\xi - \xi^2 + 2r_i\xi^2}{8(r_i - 1)^2(\xi - 1)} - \frac{\xi(r_i + \xi)}{8(r_i - \xi)} \right], \\
Q'(b) &= \frac{\xi}{(r_i - \xi)^2} \left[\frac{r_i + r_i\xi - 2\xi^2}{8(\xi - 1)^2} + \frac{r_i(r_i + \xi)}{8(r_i - \xi)} \right], \\
M_D(b) &= \frac{r_i}{r_i - \xi} \left[\frac{1}{8(r_i - 1)} - \frac{\xi}{4(r_i - \xi)} \right] + \frac{r_i}{12(\xi - 1)(r_i - 1)^2} + \frac{r_i(1 - 2\xi)}{24(r_i - 1)(r_i - \xi)(\xi - 1)} \\
&\quad + \frac{r_i\xi(3r_i - \xi)}{16(r_i - \xi)^3}, \\
P_D(b) &= \frac{r_i}{(r_i - \xi)^2} \left[\frac{-r_i^2 + \xi}{8(r_i - 1)^2} + \frac{\xi(r_i + \xi)}{8(r_i - \xi)} \right] + \frac{r_i(2r_i - 3r_i\xi + \xi)}{24(r_i - 1)^3(\xi - 1)^2} + \frac{r_i\xi(2\xi - 1)}{24(\xi - 1)^2(r_i - \xi)^2} \\
&\quad + \frac{r_i\xi(-2r_i^2 - 2r_i\xi + \xi^2)}{24(r_i - \xi)^4}, \\
Q_D(b) &= \frac{r_i\xi}{(r_i - \xi)^2} \left[\frac{1}{8(\xi - 1)} - \frac{r_i + \xi}{8(r_i - \xi)} \right] + \frac{r_i\xi}{(r_i - \xi)^2} \left[\frac{-2\xi + 1}{24(\xi - 1)^2} + \frac{2r_i^2 + 2r_i\xi - \xi^2}{24(r_i - \xi)^2} \right].
\end{aligned}$$

Appendix F. The loop functions appearing in diagram(c) are listed.

$$\begin{aligned}
M(c) &= \frac{-r_i^2 - r_i\xi + 2\xi^2 + 3r_i\xi^2 - 3\xi^3 - r_i\xi^3 + \xi^4}{24(r_i - \xi)^2(\xi - 1)^2} + \frac{-1 - 2r_i + r_i^2 + \xi + r_i\xi}{24(r_i - 1)^2(\xi - 1)^2}, \\
&\quad + \frac{\xi(-r_i^2 + 5r_i\xi + 2\xi^2)}{48(r_i - \xi)^3}, \\
P(c) &= r_i \left[\frac{3 - r_i - 2\xi}{24(\xi - 1)^2(r_i - 1)^3} + \frac{1}{24(r_i - \xi)^2(\xi - 1)^2} \right. \\
&\quad \left. + \frac{-r_i^2 + 2r_i\xi + r_i^2\xi - \xi^2 - 2r_i\xi^2 - 2\xi^3}{24(r_i - \xi)^4} \right], \\
Q(c) &= -P(c) + \frac{r_i(3 - r_i - 2\xi)}{24(\xi - 1)^2(r_i - 1)^3}, \\
M'(c) &= \frac{1}{r_i - \xi} \left[\frac{-r_i\xi - r_i + 2\xi}{8(r_i - 1)(\xi - 1)} + \frac{r_i\xi}{4(r_i - \xi)} \right], \\
P'(c) &= \frac{r_i(1 - 2\xi)}{4(r_i - \xi)(r_i - 1)(\xi - 1)} + \frac{\xi(-2\xi + 3r_i)}{8(\xi - 1)(r_i - \xi)^2} \\
&\quad + \frac{2 - r_i}{8(\xi - 1)(r_i - 1)^2} - \frac{r_i\xi(r_i + \xi)}{8(r_i - \xi)^3}, \\
Q'(c) &= \frac{\xi}{(r_i - \xi)^2} \left[\frac{r_i + r_i\xi - 2\xi^2}{8(\xi - 1)^2} + \frac{r_i(r_i + \xi)}{8(r_i - \xi)} \right], \\
M_D(c) &= \frac{1}{r_i - \xi} \left[\frac{4r_i - 6\xi + 3r_i\xi}{24(r_i - 1)(\xi - 1)} - \frac{r_i\xi}{4(r_i - \xi)} \right] + \frac{r_i\xi(3\xi + \xi^2 - 9r_i + 5r_i\xi)}{48(r_i - \xi)^3(\xi - 1)}, \\
P_D(c) &= \frac{1}{\xi - 1} \left[-\frac{(-2\xi + 1)r_i}{4(r_i - 1)(r_i - \xi)} + \frac{r_i - 2}{8(r_i - 1)^2} + \frac{\xi(-4r_i^2 + r_i^2\xi - 2\xi^2 + 4r_i\xi + r_i\xi^2)}{8(r_i - \xi)^3} \right] \\
&\quad + \frac{r_i}{(\xi - 1)^2} \left[\frac{1}{24(r_i - 1)^2} - \frac{\xi(-4r_i\xi^2 + 2r_i\xi^3 + \xi^3 + 3r_i^2 - 3r_i^2\xi + r_i^2\xi^2)}{24(r_i - \xi)^4} \right], \\
Q_D(c) &= \frac{\xi}{(r_i - \xi)^2} \left[\frac{-r_i - r_i\xi + 2\xi^2}{8(\xi - 1)^2} - \frac{r_i(r_i + \xi)}{8(r_i - \xi)} \right] \\
&\quad + \frac{r_i\xi(-4r_i\xi^2 + 2r_i\xi^3 + \xi^3 + 3r_i^2 - 3r_i^2\xi + r_i^2\xi^2)}{24(\xi - 1)^2(r_i - \xi)^4}.
\end{aligned}$$

Appendix G. The loop functions appearing in diagram(d) are listed.

$$\begin{aligned}
M(d) &= \frac{1}{24} \frac{r_i(4r_i^2 - 5r_i\xi - 5\xi^2)}{(r_i - \xi)^3}, \\
P(d) &= -Q(d) = \frac{r_i^2\xi(-r_i + 2\xi)}{4(r_i - \xi)^4}, \\
M'(d) &= \frac{-r_i(\xi + r_i)}{4(r_i - \xi)^2}, \\
P'(d) &= -Q'(d) = \frac{r_i^2\xi}{2(r_i - \xi)^3}, \\
M_D(d) &= \frac{r_i(r_i + \xi)}{4(r_i - \xi)^2} + \frac{r_i(-2r_i^2 - 5r_i\xi + \xi^2)}{24(r_i - \xi)^3}, \\
P_D(d) &= -Q_D(d) = \frac{r_i^3\xi}{4(r_i - \xi)^4} - \frac{r_i^2\xi}{2(r_i - \xi)^3}.
\end{aligned}$$

Appendix H. The loop functions appearing in diagram(a) are listed.

$$\begin{aligned}
M(a) &= \frac{8 - 35r_i + 66r_i^2 - 41r_i^3 + 2r_i^4}{24(r_i - 1)^4} - \frac{3r_i - 2}{12(r_i - 1)^2(\xi - 1)} \\
&\quad - \frac{r_i}{12(r_i - \xi)(r_i - 1)} + \frac{r_i^2 - r_i\xi - r_i^2\xi + r_i\xi^2 + \xi^3}{12(r_i - \xi)^2(\xi - 1)}, \\
P(a) &= r_i \left[\frac{-\xi}{4(\xi - 1)(r_i - \xi)(r_i - 1)} + \frac{3r_i^2}{4(r_i - 1)^4} \right. \\
&\quad \left. + \frac{3r_i - 1}{24(r_i - 1)^3(\xi - 1)} + \frac{r_i - 3\xi + 2r_i\xi - 2\xi^2}{24(\xi - 1)(r_i - \xi)^3} + \frac{1 + 5\xi}{24(r_i - \xi)^2} \right], \\
Q(a) &= \frac{\xi^2}{(\xi - 1)(r_i - \xi)^2} \left[\frac{4r_i - 3\xi - r_i\xi}{12(\xi - 1)} + \frac{r_i(-r_i + 3\xi)}{24(r_i - \xi)} \right], \\
M'(a) &= \frac{1}{r_i - 1} \left[\frac{r_i\xi^2 - \xi^2 + r_i - \xi}{4(\xi - 1)(r_i - \xi)} + \frac{3(3r_i - 1)}{4(r_i - 1)} \right], \\
P'(a) &= \frac{r_i^2}{(1 - r_i)^2} \left[-\frac{r_i(\xi - 1)}{4(r_i - \xi)^2} + \frac{3}{2(1 - r_i)} \right], \\
Q'(a) &= \frac{1}{4} \frac{\xi^2(-3r_i + 2\xi + r_i\xi)}{(\xi - 1)^2(r_i - \xi)^2}, \\
M''(a) &= \frac{11 - 13r_i}{4(r_i - 1)^2}, \\
P''(a) &= \frac{r_i(-5 + 6r_i)}{2(r_i - 1)^3}, \\
M_D(a) &= \frac{3r_i - 1}{8(r_i - 1)^2} + \frac{2r_i^2 + 5r_i - 1}{6(r_i - 1)^3} - \frac{\xi^2}{4(\xi - 1)(r_i - \xi)} \\
&\quad + \frac{r_i}{\xi - 1} \left[\frac{-1 + 7r_i - 5r_i\xi + \xi - 2r_i^2\xi}{24(r_i - 1)^3} + \frac{\xi}{12(r_i - \xi)} + \frac{\xi^2}{12(r_i - \xi)^2} \right], \\
P_D(a) &= -\frac{r_i^2}{4(r_i - 1)^3} - \frac{r_i^2}{(r_i - 1)^4} + \frac{\xi(-r_i + \xi - r_i\xi)}{4(\xi - 1)(r_i - 1)(r_i - \xi)} + \frac{(2r_i - \xi)\xi^2}{4(r_i - \xi)^2(\xi - 1)} \\
&\quad + \frac{r_i}{\xi - 1} \left[\frac{-4r_i - 3r_i^2 + 1 + 6r_i^2\xi}{24(r_i - 1)^4} - \frac{\xi^2}{12(r_i - \xi)^2} - \frac{\xi^2(r_i + \xi)}{24(r_i - \xi)^3} \right], \\
Q_D(a) &= \frac{r_i\xi^2}{12(\xi - 1)(r_i - \xi)^2} + \frac{r_i\xi^2(r_i + \xi)}{24(\xi - 1)(r_i - \xi)^3} - \frac{\xi^2(-2r_i + r_i\xi + \xi)}{4(r_i - \xi)^2(\xi - 1)^2}.
\end{aligned}$$

Appendix I. The loop functions appearing in diagram Z are listed.

$$\begin{aligned}
MM(b) &= \frac{7 - 3r_\alpha + 6r_\alpha^2 - 28r_\alpha^3}{12(r_\alpha - 1)^3} + \frac{30r_\alpha^3 - 22r_\alpha^2\xi - r_\alpha\xi^2 - \xi^3}{12(r_\alpha - \xi)^3}, \\
PP(b) &= \frac{r_\alpha^2(-5 + 8r_\alpha)}{2(r_\alpha - 1)^4} - \frac{r_\alpha\xi(24r_\alpha^2 - 25r_\alpha\xi + 10\xi^2)}{6(r_\alpha - \xi)^4}, \\
QQ(b) &= \frac{r_\alpha\xi(24r_\alpha^2 - 25r_\alpha\xi + 10\xi^2)}{6(r_\alpha - \xi)^4}, \\
MM'(b) &= \frac{-4 - 5r_\alpha + 3r_\alpha^2}{2(r_\alpha - 1)^2} + \frac{r_\alpha(-3r_\alpha + \xi)}{2(r_\alpha - \xi)^2}, \\
PP'(b) &= \frac{1 - 3r_\alpha - 3r_\alpha^2 - r_\alpha^3}{2(r_\alpha - 1)^3} + \frac{r_\alpha^3 + r_\alpha^2\xi + r_\alpha\xi^2 - \xi^3}{2(r_\alpha - \xi)^3}, \\
QQ'(b) &= -\frac{r_\alpha\xi(-2r_\alpha + \xi)}{(r_\alpha - \xi)^3}, \\
MM_D(b) &= -\frac{5 + 7r_\alpha - 12r_\alpha^2 + 6r_\alpha^3}{4(r_\alpha - 1)^3} + \frac{18r_\alpha^3 - 22r_\alpha^2\xi + 11r_\alpha\xi^2 - \xi^3}{12(r_\alpha - \xi)^3}, \\
PP_D(b) &= \frac{-1 + 4r_\alpha - 9r_\alpha^2 + 4r_\alpha^3 - r_\alpha^4}{6(r_\alpha - 1)^4} + \frac{3r_\alpha^4 - 12r_\alpha^3\xi + 21r_\alpha^2\xi^2 - 10r_\alpha\xi^3 + \xi^4}{6(r_\alpha - \xi)^4}, \\
QQ_D(b) &= \frac{r_\alpha(-2r_\alpha^3 + 8r_\alpha^2\xi - 15r_\alpha\xi^2 + 6\xi^3)}{6(r_\alpha - \xi)^4}, \\
MM(c) &= \frac{r_\alpha(-29r_\alpha^2 + 19r_\alpha\xi + 4\xi^2)}{12(r_\alpha - \xi)^3}, \\
PP(c) = -QQ(c) &= \frac{r_\alpha\xi(24r_\alpha^2 - 25r_\alpha\xi + 10\xi^2)}{6(r_\alpha - \xi)^4}, \\
MM'(c) &= \frac{r_\alpha(3r_\alpha - \xi)}{2(r_\alpha - \xi)^2}, \\
PP'(c) = -QQ'(c) &= \frac{r_\alpha\xi(-2r_\alpha + \xi)}{(r_\alpha - \xi)^3}, \\
MM_D(c) &= -\frac{r_\alpha(17r_\alpha^2 - 19r_\alpha\xi + 8\xi^2)}{12(r_\alpha - \xi)^3}, \\
PP_D(c) = -QQ_D(c) &= \frac{r_\alpha(-2r_\alpha^3 + 8r_\alpha^2\xi - 15r_\alpha\xi^2 + 6\xi^3)}{6(r_\alpha - \xi)^4}.
\end{aligned}$$

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